A. C. True, of Washington, D. C., is the dean of the school, and the assistant dean is Professor Charles E. Marshall, director of the graduate school and professor of microbiology at the Massachusetts College. The school is open to all college graduates. Its purpose is for the study of the recent development in the natural, social and economic sciences as applied to agriculture, as well as in the technical branches of the so-called practical agriculture. Courses are offered in (1) growth, (2) production, (3) rural organization, (4) agricultural education, (5) distribution-marketing, (6) land problems, (7) adjunct course in physico-chemico-physiological elements, (8) special lectures and conferences.

FROM May 1 to November 30, fourth-year medical students at Toronto will be given a summer course to qualify for their degree, and there will be no fifth-year course next year at the university. The summer session will last twenty-six weeks. There will be sixty men who will attend the summer session, and when they graduate they will be in a position to accept positions with the various hospital units. It is understood that Queen's University, Kingston, will take a similar step.

THE trustees of Northwestern University at their last meeting filled the vacancy in the deanship of the dental school, which occurred through the death of Dr. Greene Vardiman Black on August 31 of last year, by the election of Thomas Lewis Gilmer, M.D., D.D.S., Sc.D.

AT Smith College, Dr. Joel E. Goldthwaite has been appointed professor of hygiene and physical education, and Miss Pauline Sperry, assistant professor of mathematics. Miss Harriet R. Cobb has been promoted to be professor of mathematics; Dr. Mary M. Hopkins to be associate professor of astronomy, and Mrs. Anna B. Newell to be assistant professor of zoology.

AT Harvard University Dr. Dunham Jackson has been promoted to an assistant professorship of mathematics.

THE chair of botany in the Alabama Polytechnic Institute and Agricultural Experiment Station, vacant by the resignation of Dr. J. S. Caldwell to take up the position of "By-Products Specialist" for the Washington Agricultural Experiment Station, has been filled by the appointment of W. J. Robbins, Ph.D. (Cornell), instructor in botany in the New York State College of Agriculture.

DISCUSSION AND CORRESPONDENCE

THE FUNDAMENTAL EQUATION OF MECHANICS

MR. KENT'S recent letter on the "Teaching of Elementary Dynamics,"[1] with much of which I heartily agree, contains one serious error which I think should not pass unnoticed. As this error seems to me a not unnatural result of one feature of his favorite method of beginning the study of mechanics, I should like to take this opportunity to summarize, in a brief review, the three methods of beginning mechanics which have been advocated respectively by Mr. Kent, Professor Hoskins and myself.[2] To do this, I propose first to state briefly certain *dynamical principles on which we all agree;* I shall then endeavor to show that precisely these non-disputed facts *are all that the student needs to know in order to solve dynamical problems, provided he follows my method.* It is only when he endeavors to follow one of the other methods that he is led into controversial territory. If undisputed facts are sufficient for the solution of problems, why burden the student's mind (except as a matter of historical interest) with needless disputations?

I. The following statements will, I believe, be accepted as true by all of us, though the emphasis placed on the various items would doubtless vary.

1. A *force* is a familiar notion which may be thought of as a push or a pull. Any given force may be *identified,* that is, preserved for

[1] SCIENCE, December 24, 1915.

[2] See articles in SCIENCE by L. M. Hoskins, December 4, 1914, April 23, May 7, August 27 and September 10, 1915; by E. V. Huntington, February 5 and July 30, 1915; and by William Kent, March 19 and December 24, 1915.

reference, by noting the distance it compresses an (idealized) standard spring.

2. To *measure* a force, we require a *unit* force, and a *scale* of multiples and submultiples of that unit. Such a scale can be readily constructed by simply opposing one or more springs, in various combinations, against the standard, or unit, spring. (This process of calibrating a spring balance does not involve any assumption in regard to Hooke's Law; it requires merely that the elastic properties of the spring, whatever they may be, do not, in the ideal case, vary with the time.)

3. A *material body,* or lump of matter, is a familiar notion, in the sense that if any material is added to or taken away from the body, it ceases to be the same body. (In this discussion, "body" is used in the sense of "particle," that is, a body which may be supposed, for the purpose in hand, to be concentrated at a single point.)

4. A *motion* of a body, with respect to a given frame of reference, is also a familiar idea. The scientific concepts of *velocity* and *acceleration* serve merely to make quantitatively precise our qualitative notions of "faster" and "slower."

5. The *effect of a force* when applied to a body free to move, is to *change the velocity* of the body. As a matter of common observation, the force required to produce a given change of velocity in a given time is larger for some bodies than for others.

6. *If a given body is acted on, at two different times, by two forces, F and F', and if a and a' are the corresponding accelerations, then*

$$F/F' = a/a';$$

that is, in the case of any given body, *the accelerations are proportional to the forces.* This statement is best regarded as a scientific hypothesis, the consequences of which have been abundantly verified by experiment.

7. In order to *predict the behavior of any given body* under the action of various forces (and this is the central problem of dynamics), it is sufficient, and necessary, to know from some (direct or indirect) experiment, what acceleration some one force would produce in that body. The acceleration, a, that would be produced by any other force can then be computed at once by the fundamental proportion.

In the special case in which the acceleration a is constant, and the body starts from rest, $v = at$ and $x = \frac{1}{2}at^2$; in the general case, v and x must be obtained from a by integration.

The foregoing items 1–7 are quite general;[3] the following, 8–11, are suggested primarily by observations on the earth's surface.

8. The observed acceleration, g, of a freely falling body, in any locality, is the same for all bodies. By the "standard locality" is meant any locality (for example, approximately 45° latitude, sea level) in which $g = g_0 = 980.665$ cm./sec.^{2} $= 32.1740$ ft./sec.^{2}, this being the convention now generally adopted.[4]

9. (*a*) *The force required to support a body at rest with respect to the earth in the standard locality,* and (*b*) *the force which would give that body, if free to move* (in any locality) *the standard acceleration g_0, are the same.* This force, which is characteristic of the given body, is what I have called the *standard weight,* W_0, of the body.[5] By the fundamental proportion, if F is any other force, and a the corresponding acceleration, then $F/W_0 = a/g_0$.

10. The "standard weight" of a body can always be found (in any locality) by the familiar process of "*weighing*" *the body on a beam balance.*

For example, suppose a given body balances

[3] The principle of action and reaction, the principle of the vector addition of forces and the principle of the independence of two perpendicular forces, together with the definitions of such terms as work, kinetic energy, impulse, momentum, etc., although necessary for the development of the science, may be passed over without comment, as they are not now in dispute.

[4] International Conference on Weights and Measures, Procès-Verbaux des Séances, page 172, Paris, 1901; U. S. Bureau of Standards, Circular No. 34, second edition, page 6, 1914.

[5] SCIENCE, July 30, 1915, page 161. A defect in my earlier form of the definition (*Bulletin of the Society for the Promotion of Engineering Education,* June, 1913; compare U. S. Bureau of Standards, *loc. cit.*, page 7) was called to my attention by Professor Hoskins's criticism in SCIENCE, April 23, 1915.

a "1 lb. weight" in a given locality; this means, primarily, that the force required to support the given body, in the given locality, is equal to the force required to support the "1 lb. weight" in that locality; hence, by a simple inference, the force required to support the given body in the standard locality will be equal to the force required to support the "1 lb. weight" in the standard locality; but the force required to support the "1 lb. weight" in the standard locality is guaranteed to be "1 lb."; hence the force required to support the given body in the standard locality—that is, the "standard weight" of the body—is also 1 lb.

11. Finally, *if the standard weight of a body is known, the dynamical properties of the body are wholly determined.* For example, if we wish to find the acceleration, a, produced by any force F in a body whose standard weight is W_0, we have merely to substitute the given values in the equation $F/W_0 = a/g_0$ and solve for a.

In other words, the simple principles enumerated above—principles the truth of which has not been called in question—form a complete and satisfactory foundation for the solution of elementary problems in dynamics. It should be particularly noted that no restrictions whatever are imposed on the choice of the fundamental units of force, length and time; and that the only datum that we need to know in advance concerning any body that enters a problem is a single, readily determined force, namely, the standard weight of the body.

II. Let us now turn to Professor Hoskins's method, and inquire what items one of the very best of the modern text-book writers regards it as necessary to add to these familiar principles.

In his article in Science for April 23, 1915, page 608, he says:

> The method most intelligible to the beginner is to introduce at the outset the body-constant which was called by Newton mass or quantity of matter, and to make the fundamental principle . . . the following: (a) A force acting upon a body otherwise free would give it, at every instant, an acceleration proportional directly to the force and inversely to the mass of the body.

His fundamental equation is therefore

$$\frac{a}{a'} = \frac{F}{F'} \cdot \frac{m'}{m},$$

which is immediately thrown, by a perfectly arbitrary restriction on the choice of units, into the final form: $F = ma$.

It will be noticed that this method of Professor Hoskins, and most other authors, involves four fundamental concepts, namely, force, length, time and mass; while my method involves only three, namely, force, length and time. My chief objection to this complication is not merely that the fourth concept, mass, is superfluous, *as a fundamental concept,* but also that this concept is, *at this stage,* exceedingly ill-defined. Nowhere in Professor Hoskins's papers can one find a clear-cut statement of what he really intends the student, at the outset, to understand by mass. If he means merely that m is a quantity proportional to F/a, and m' a quantity proportional to F'/a', which relations are consistent with his fundamental equation, of course no one could object to this use of symbols; but the compound quantity F/a, or W/g, which is properly called the *inertia* of the body, can surely not be understood "at the outset," before the elements out of which it is built up have been grasped; and this is clearly not Professor Hoskins's intention.

His several attempts to point out certain rather vague analogies between the concept of mass or inertia and certain other concepts,[6] have altogether failed to provide a satisfactory justification for his use of mass as a term the meaning of which can be presupposed at the outset, for nowhere does he really define the term, and nowhere does he squarely meet the objections which I have raised to this procedure.

A further objection to the equation $F = ma$, which I have dwelt upon at length elsewhere,[7] is in regard to the question of units. The choice of units which the use of this equation compels is needlessly complicated and quite

[6] L. M. Hoskins, Science, September 10, 1915.

[7] See especially Science, July 30, 1915, page 160.

unscientific. On this point I heartily endorse Mr. Kent's somewhat pungent criticisms, which are entirely in accord with previous expressions of my own.

III. Finally, let us examine Mr. Kent's method. Like Professor Hoskins, Mr. Kent introduces mass, or quantity of matter, at the outset, but unlike Professor Hoskins, he frankly defines what he means by this term—namely, the result of weighing on a beam balance. This same plan is followed by several of the writers who have taken part in this discussion, notably by Franklin and MacNutt.[8] There is no logical objection to this procedure; but—*cui bono?* The result of weighing a body on a beam balance gives primarily the standard weight of the body as we have seen in (10), above; if the standard weight of a body, which is simply a force, and which everybody understands, is all that is needed, why rename this familiar concept by a less familiar term, like "quantity of matter"? Further, what are the "dimensions" of "quantity of matter"?[9] Is it of the nature of a force, or, like inertia, of the nature of a force divided by an acceleration? Or is it wholly independent of force, length and time (in which case it is wholly superfluous)? Without doubt, *later in the course* any terms of this sort may be introduced at pleasure; but why confuse the *beginner* with any concepts that are not really needed? The elimination of this one term, quantity of matter, would bring Mr. Kent's method almost exactly into line with my own, except for one point.

This remaining point of difference, though slight, is, at least from the point of view of the teacher, an important one. My method begins frankly with the idea of acceleration as a fundamental concept—not an easy idea, but one which is so essential that the student does best who faces its difficulty squarely at the outset. Mr. Kent's method, on the other hand, begins with the comparatively simple special case in which the acceleration is constant, and introduces the real thing only later on, as a sort of afterthought. For a student who is never to go further than the special case, the method based on Mr. Kent's equation $V = FTg/W$ is well enough; and this is what I had in mind when I said that "the method was not without interest on the pedagogic side." But for the student who pursues the subject seriously, the plan of spending so much time on the simple special case is too apt to have only one result, namely that the development of an unerring, instinctive grasp of what acceleration really means is either long delayed or never attained. And this brings me to the question of the error in Mr. Kent's paper. On page 902 he asks the question:

> How can a body at rest on the earth's surface have an acceleration . . . radially toward the earth's center . . . if there is no change in the speed of rotation of the earth?

The old error of supposing that a particle moving with constant velocity in a curved path has no acceleration!

Now I have no doubt that if the matter were called to his attention, Mr. Kent would at once remember that a particle moving with a constant velocity in a circular path certainly does have an "acceleration radially toward the center," the value of which is v^2/r; but the point I am making is that in the rush of the moment he did *not* remember this most cardinal fact about accelerated motion; and I attribute the possibility of a man of his experience making a slip of this kind entirely to the grudging fashion in which the subject of acceleration was probably presented to him in his first course in mechanics—a precedent which my method refuses to follow.

EDWARD V. HUNTINGTON
HARVARD UNIVERSITY

[8] Franklin and MacNutt, SCIENCE, July 9 and September 24, 1915. In regard to the supposedly contrasted statements (*a*) and (*b*) on page 423 of their second article, it may be remarked that these authors have apparently overlooked the fact that each of these statements is a direct mathematical consequence of the other, as one may readily see by an inspection of their diagram on page 422.

[9] Mr. Kent is apparently quite oblivious of the value of the theory of dimensions, as he uses the same letter (g) quite indiscriminately to denote a length, a velocity, an acceleration, or a pure number!